Key for Drake Physics Prize Test 2013 (Multiple Choice)

1 e
2 e
3 b
4 e
5 d
6 a
7 a
8 d
9 a
10 e
11 d
12 d
13 b
14 c
15 c
16 a
17 b
18 b
19 d
20 e
21 c
22 b
23 a
24 c
25 d

Physics Prize 2013 (Part I)

1. Which of the following is an accurate statement?

a) The magnitude of a vector can be less than the magnitude of one of its components.
b) If the magnitude of vector A is less than that of vector B, the x-component of A is smaller than the x-component of B.
c) The magnitude of a vector can be positive or negative.
d) The x-component of a vector is either zero or positive.
e) A vector cannot have zero magnitude if one of its components is not zero.

2) A centripetal force needs to act on an object to make it go on a circular path. If the angular speed is constant, does this force do any work?

a) Yes, since a force acts and the object moves.
b) Yes, since it takes energy to move an object.
c) No, because the object has constant lateral velocity.
d) No, because the force and the displacement of the object are in opposite directions.
e) No, because the force and the change in displacement of the object are perpendicular to each other.

3) If a constant torque is applied to an object, the object will

a) rotate with constant angular speed
b) rotate with constant angular acceleration
c) decrease its moment of inertia
d) increase its moment of inertia
e) do none of the above

4) A steel ball sinks in a pool of water but floats in a pool of liquid mercury. Assume that the temperature and density of the water and the mercury are constant in the respective pools. Which of the following statements is correct?

a) There is no buoyant force when the ball floats in mercury.
b) The buoyant force in the water is decreasing while the ball is sinking.
c) The buoyant force in the water is increasing while the ball is sinking.
d) The buoyant forces on the ball in the mercury and in the water are the same.
e) The buoyant force on the ball in the mercury is bigger than in the water.

5) A container holds oxygen and helium gases at the same temperature. The mass of an oxygen molecule is approximately eight times the mass of a helium atom.

$\bar{v} \propto v_{rms} \propto \sqrt{\frac{T}{m}}$

a) The oxygen molecules have the greater average kinetic energy.
b) The oxygen molecules have the greater average speed.
c) The helium atoms have the greater average kinetic energy.
d) The helium atoms have the greater average speed.
e) The average speed of the oxygen molecules and the helium atoms is the same.

6) Consider the three vectors, $\mathbf{V}_1 = (3,1)$, $\mathbf{V}_2 = (2,4)$, and $\mathbf{V}_3$ (-3,4). The length of the vector $\mathbf{V}_3 + \mathbf{V}_2 - \mathbf{V}_1$ is about

a) 8.1
b) 6.4
c) 2.2
d) 1.7
e) 3.0

$3+2-(-3) = 8$, $1+4-4 = 1$ $\Rightarrow$ length $= \sqrt{8^2+1^2} = \sqrt{65} \approx 8.1$

7) A car's speed decreases from 60 m/s to 40 m/s over a distance of 200 m. The average acceleration is about

a) $-5.0\ m/s^2$ ✗
b) $-9.6\ m/s^2$
c) $-12.5\ m/s^2$
d) $-18.2\ m/s^2$
e) $-23.1\ m/s^2$

average speed is $50\ \frac{m}{s}$ ⇒ it happens over 4 seconds

$\Rightarrow \frac{-20\ m/s}{4\ s} = -5\ m/s^2$

Or: $v^2 = v_0^2 + 2ad \Rightarrow a = \frac{v^2 - v_0^2}{2d} = \frac{1600-3600}{400} \cdot \frac{m}{s^2} = -5\ \frac{m}{s^2}$

8) A swinging simple pendulum reaches a maximum height of 0.12 m above its equilibrium position (h = 0). At the point where the kinetic and potential energy (with respect to the equilibrium position) are the same, the height of the pendulum bob is about

a) 0.110 m
b) 0.090 m
c) 0.085 m
d) 0.060 m ✗
e) 0.030 m

$P.E. = \frac{1}{2} P.E._{max} = \frac{1}{2} mgh_{max} \Rightarrow h_{max} = 0.06\ m$

9) Increasing a tone by one octave corresponds to doubling the frequency. Suppose the string of a grand piano for playing a certain note is 2 m long. If the tension were the same in all strings, approximately how long would the string be for the note three octaves higher?

a) 25 cm ✗
b) 50 cm
c) 67 cm
d) 80 cm
e) 100 cm

$\lambda \propto L$, $f \propto \frac{1}{\lambda}$ if v is constant (✓ since same tension)

$\Rightarrow f \to 8f$ requires $\lambda \to \lambda/8$, $L \to L/8$

10) A 0.50 kg mass is attached to the end of a 1.0 m string. The system is whirled in a nearly horizontal circular path. If the maximum tension that the string can withstand is 350 N, what is the approximate maximum speed of the mass for the string not to break?

a) 5 m/s
b) 8 m/s
c) 12m/s
d) 18 m/s
e) 26 m/s ✗

$F_c = \frac{mv^2}{r} \Rightarrow v_{max} = \sqrt{\frac{F_{max} \cdot r}{m}} \approx 26\ m/s$

11) Equal masses of ice at a temperature of 0 °C and steam at 100 °C are brought together. The specific heat of liquid water is 1.0 cal per gram and °C, and the latent heats for fusion and vaporization are 80 cal/g and 540 cal/g, respectively. When thermal equilibrium is reached, the temperature of the system is about

a) 0 °C;
b) 50 °C
c) 87 °C;
d) 100 °C ✗
e) unknown, since the mass is not given

Take just 1 g of steam. We need to lose 540 cal to liquify the entire gram of steam. That's more than the 180 cal needed to melt the ice and then heat the water up to 100 °C. ⇒ 100 °C (mixture of steam and liquid)

12) A mass hanging from a spring with negligible mass is oscillating up and down. When a second identical mass is added to the first one,

a) the frequency of the oscillations increases by a factor of about 2
b) the frequency of the oscillations increases by a factor of about 1.4
c) the frequency of the oscillations remains unchanged
~~d)~~ the frequency of the oscillations decreases by a factor of about 1.4
e) the frequency of the oscillations decreases by a factor of about 2

$\omega = \sqrt{\frac{k}{m}} = 2\pi f$

$\Rightarrow m \to 2m \hat{=} f \to \frac{f}{\sqrt{2}}$, $\sqrt{2} \approx 1.4$

13) In an introductory lab, you measure the length and width of a rectangle as 1.025 m and 0.63 m, respectively. You should report the area of that rectangle as

a) 0.6 m^2
~~b)~~ 0.65 m^2
c) 0.646 m^2
d) 0.6458 m^2
e) 0.64575 m^2

$1.025 \cdot 0.63 = 0.6475 \sim 0.65$ with two significant figures

14) A particular organ pipe can resonate at 264 Hz, 440 Hz, and 616 Hz, but not at any other frequency between those given above. Consequently, 616 Hz is what multiple of the fundamental frequency?

a) 2
b) 4
~~c)~~ 7
d) 9
e) 11

There will also be a resonance frequency of 264 Hz − 176 Hz = 88 Hz

$\Rightarrow \frac{616}{88} = 7$

15) A stiff, nonflammable rod is used to push burning logs around in a fireplace. The ideal material of the poker should have

a) a high specific heat and high thermal conductivity
b) a low specific heat and low thermal conductivity
~~c)~~ a high specific heat and low thermal conductivity
d) a low specific heat and high thermal conductivity
e) a low specific heat and high mass density

We want a lot of heat to warm up the poker and a small heat flow

16) Which of the following statements for an adiabatic expansion of an ideal gas is true?

~~a)~~ No heat is added to the gas.
b) The temperature remains constant.
c) Work is done on the gas.
d) Both a) and b) are true.
e) All of a), b), and c) are true statements.

17) Consider two spherical planets with a homogeneous mass distribution. Planet A has twice the mass and twice the radius of planet B. The ratio of gravitational forces experienced by objects on the surface of planet A over that experienced by objects on the surface of planet B is about

a) 1/4
~~b)~~ 1/2
c) 1
d) 2
e) 4

$mg = G \cdot \frac{m \cdot M_{planet}}{R_{planet}^2} \Rightarrow \frac{g_A}{g_B} = \frac{M_A / M_B}{R_A^2 / R_B^2} = \frac{2}{4} = \frac{1}{2}$

18) A copper wire is found to break when exposed to a minimum tension of 36 N. A similar wire with half the diameter is expected to break at a minimum tension of

a) 6 N
~~b)~~ 9 N
c) 18 N
d) 36 N
e) 54 N

stress = $\frac{\text{force}}{\text{area}}$ => diameter $\times \frac{1}{2}$ => area $\times \frac{1}{4}$

=> $\frac{1}{4}$ of the force causes the same stress

19) A very large object moving with velocity **V** collides head-on with a very small object moving with velocity –**V**. The collision is elastic, and there is no friction. The large object barely slows down. What is the speed of the small object after the collision?

a) nearly zero
b) nearly V
c) nearly 2 V
~~d)~~ nearly 3 V
e) nearly 5 V

$\vec{V}_{rel}$ (before) = $\vec{V} - (-\vec{V}) = 2\vec{V}$

$= -V_{rel}$ (after) => the small object will have a velocity of $\approx 3\vec{V}$

20) The quantity force times distance over time corresponds to

a) the kinetic energy of an object
b) the potential energy of an object
c) the acceleration of an object
d) the work done on an object
~~e)~~ the power given to an object

21) Which of the following statements is correct?

a) A particle's speed and velocity can be in the same direction.
b) A particle's angular momentum and velocity are in the same direction.
~~c)~~ A particle's velocity and acceleration can be in opposite directions.
d) The kinetic energy of a particle is the same as the kinetic energy of its center of mass.
e) If a force is applied to a particle, the kinetic energy will increase.

22) A canon ball is launched horizontally from a cliff with a speed of 100 m/s. Three seconds later, the ball hits the ground. The approximate height of the cliff is

a) 30 m
~~b)~~ 45 m
c) 90 m
d) 150 m
e) 300 m

only the vertical motion matters; $y = -\frac{1}{2}gt^2 \approx -45\,m$

(if $g = 10\,\frac{m}{s^2}$)

23) A boy and a girl, both of the same mass and general body shape, are riding a merry-go-round. The boy sits at the outer edge of the merry-go-round, whereas the girl sits half way between the center and the edge. Which of the following statements is correct?

~~a)~~ Both have the same angular speed, but the boy has the larger linear speed.
b) Both have the same angular speed, but the girl has the larger linear speed.
c) Both have the same linear speed, but the boy has the larger angular speed.
d) Both have the same linear speed, but the girl has the larger angular speed.
e) Both have the same angular and linear speed.

24) Three masses, $m_1 = 10$ kg, $m_2 = 20$ kg, and $m_3 = 40$ kg are located at the following locations on a straight line: m_1 at $= -2.0$ m; m_2 at +3.0 m; m_3 at + 5.0 m. Consequently, the center of mass of this system of three masses is located at

a) 0.0 m
b) 1.8 m
~~c)~~ 3.4 m
d) 4.0 m
e) 5.0 m

$$x_{cm} = \frac{10 \cdot (-2) + 20 \cdot 3 + 40 \cdot 5}{70} \, m \approx 3.4 \, m$$

25) Compared to yesterday, you did three times the work in one third of the time. To do so, your power output must have been

a) the same as yesterday
b) three times that of yesterday
c) one third of yesterday's
~~d)~~ nine times that of yesterday
e) one ninth of yesterday's

power = work / time

=> if work × 3 and time × 1/3
then power × 9

Drake Physics Prize Exam 2013 ... Part 2

Name: **School:** **Proctor:**

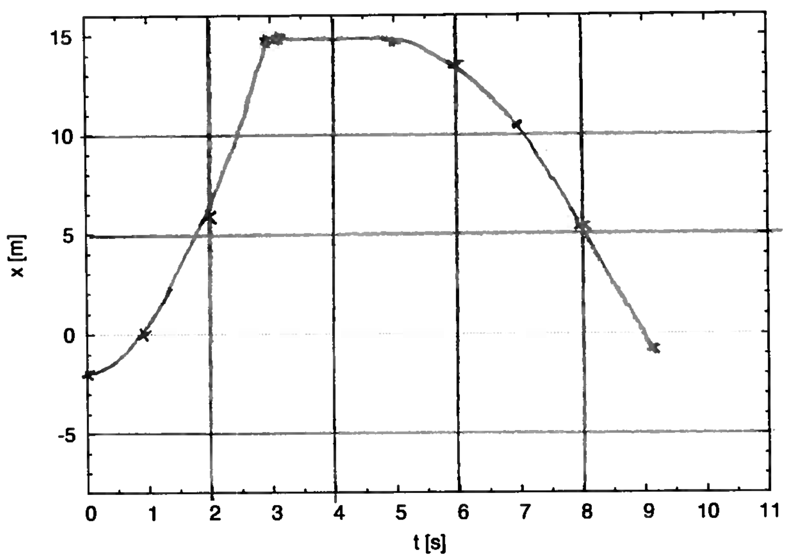
1. A particle starts at $t = 0.0\,\mathrm{s}$ from rest at the position $x = -2.0\,\mathrm{m}$. For the first two seconds, it is accelerated with $a = 4.0\,\mathrm{m/s^2}$ before moving on for another second with constant speed. It then comes to a quick stop (within approximately 0.10 s). After doing nothing for the next 1.9 s, the particle accelerates for three seconds with $a = -2.0\,\mathrm{m/s^2}$ before continuing with constant speed for another second.

   a) Draw the curve into the panel below that shows the particle's position as a function of time for the entire nine seconds described above. (5 pts)

$x(t) = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$

$v(t) = v_0 + a(t - t_0)$

$x(1s)$
$x(2s)$
$x(3s)$
$x(3.1s)$
$x(6s)$
$x(7s)$
$x(8s)$
$x(9s)$



   b) What is the average velocity over the particle's entire journey? (3 pts)

$$\vec{v}_{ave} = \frac{-0.6\,m - (-2.0\,m)}{9\,s} = \frac{1.4\,m}{9\,s} \approx 0.15\,\frac{m}{s}$$

   c) What is the average speed over the particle's entire journey? (3 pts)

$$|\vec{v}_{ave}| = \frac{16.4\,m + 15\,m}{9\,s} \approx 3.5\,\frac{m}{s}$$

   d) What is the biggest magnitude of acceleration experienced by the particle, when does it happen, and what is the direction of the acceleration at that time? (4 pts)

It happens during the 0.1s when the particle is brought to rest from a velocity of then $8\,\frac{m}{s}$

$$\vec{a}_{max} = \frac{0\,\frac{m}{s} - 8\,\frac{m}{s}}{0.1\,s} \approx -80\,\frac{m}{s^2}$$

2. A motorcyle show sometimes features a driver going (at a constant height above ground) around the vertical walls of a cylinder, apparently beating the gravitational force.

a) What is the angular speed (in rad/s) of the motorcycle for a tangential speed of 36 km/h and a cylinder radius of $r = 5\,\mathrm{m}$? (3 pts)

$$\omega = \frac{v}{r} = \frac{36{,}000\ \mathrm{m} / 3{,}600\ \mathrm{s}}{5\,\mathrm{m}} = 2\ \frac{\mathrm{rad}}{\mathrm{s}}$$

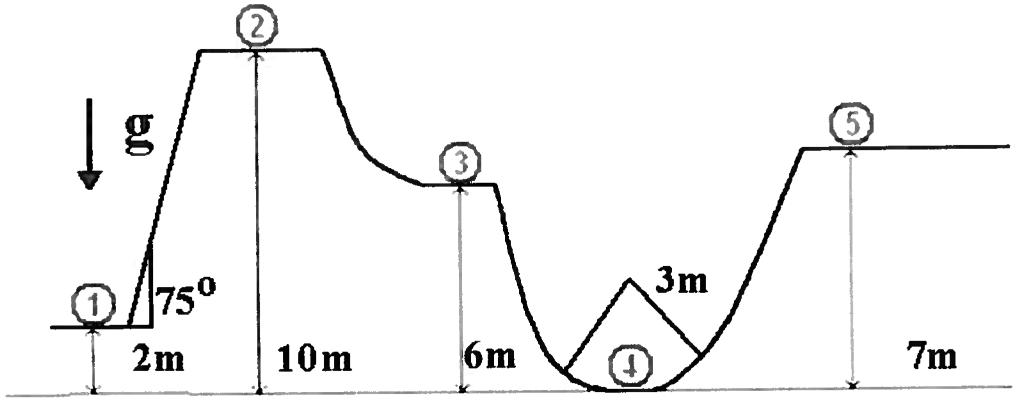
b) What is the centripetal acceleration (in $\mathrm{m/s^2}$) felt by the driver in this case? (3 pts)

$$a = \frac{v^2}{r} = \frac{(10\,\mathrm{m/s})^2}{5\,\mathrm{m}} = 20\ \mathrm{m/s^2} \approx 2g$$

c) What is the minimum coefficient of friction required between the motorcycle's tires and the cylinder's wall to make this act possible? (5 pts)

$$F_{gravity} = mg \le F_{friction} = \mu m \frac{v^2}{r}$$

$$\Rightarrow \mu \geq 0.5$$



3. The figure above shows various positions of a roller coaster of mass 800 kg. Neglect friction in this problem and assume that the coaster will start its downward journey from rest at the edge of the platform with point 2.

a) How much work is required to bring the roller coaster from point 1 to point 2? (3 pts)

$W = \Delta P.E. = 800\,kg \cdot \frac{10\,m}{s^2} \cdot 8\,m \approx 64{,}000\,J$

b) What is the speed of the roller coaster at point 3? (3 pts)

$\underbrace{\frac{1}{2} m v^2}_{K.E.\ gained} = \underbrace{m g \Delta h}_{P.E.\ lost} \Rightarrow v = \sqrt{\frac{10\,m}{s^2} \cdot 2 \cdot 4\,m} \approx 9\,m/s$

c) What is the total weight by which a person of mass 75 kg is pressed into her/his seat at point 4? Assume that the part of the track near that point is circular with a radius of 3 m. (6 pts)

Speed at ④ is $\sqrt{10\,\frac{m}{s^2} \cdot 2 \cdot 10\,m}$

$a_{centripetal} = \frac{v^2}{r} = \frac{20\,meters \cdot g}{3\,m}$

$\Rightarrow$ total weight $= 75\,kg\,[g + \frac{20}{3} g] \approx 5{,}700\,N$

↑ Earth gravity

4. While leaving the station on a train, a person hears a frequency of 540 Hz coming from a siren located at the station. The actual frequency emitted by the siren, however, is 600 Hz, and the speed of sound is 340 m/s.

a) Determine the speed of the train. (4 pts)

$$f' = f \frac{v \pm v_o}{v \mp v_s}$$

(observer: v_o; sound: v; source: v_s)

here $540 = 600 \left[1 - \frac{v_o}{v} \right]$

$\Rightarrow \frac{v_o}{v} = 0.1 \Rightarrow v_o = 34 \frac{m}{s}$

b) Determine the frequency of the siren that would be heard by a person at the station if the siren were on a train approaching the station with the speed determined in a). (2 pts)

$$f' = 600\,Hz \cdot \frac{1}{1 - \frac{v_s}{v_o}} = \frac{600\,Hz}{0.9} \approx 670\,Hz$$

5. A 0.150 kg sample of an alloy is heated to 540 °C and quickly dropped into 0.400 kg of water at 10 °C, which is contained in a 0.200 kg calorimeter cup made of aluminum. The final temperature of the mixture is 30 °C. The heat capacity of water is 4, 186 J/(kg · °C) and the heat capacity of aluminum is 900 J/(kg · °C). Determine the heat capacity of the alloy. (6 pts)

heat lost by alloy = heat gained by water + cup

$$0.150\,kg \cdot c_{alloy} \cdot 510^\circ C = \left[0.400\,kg \cdot 4{,}186 \frac{J}{kg\,^\circ C} + 0.200\,kg \cdot 900 \frac{J}{kg\,^\circ C} \right] \cdot 20^\circ C$$

$$\Rightarrow c_{alloy} = \frac{[0.400 \cdot 4186 + 0.200 \cdot 900] \cdot 20}{0.150 \cdot 510} \frac{J}{kg\,^\circ C}$$

$$\approx 485 \frac{J}{kg\,^\circ C}$$