Drake Physics Prize Test 2015 (Part II)

SOLUTIONS

STUDENT: **SCHOOL:** **PROCTOR:**

Please, circle your answers among the multiple choices and write down all your solutions in detail. If you need more paper, please, indicate the problem number you are solving, write your name and school name on each page and staple all your papers together.

1. A block of mass $m = 2$ kg is attached to a spring with spring constant $k = 1.70 \times 10^3$ N/m, and rests on a frictionless, horizontal surface. The block is pushed such that that the spring compresses by a distance $x = 6$ cm from the equilibrium position, and released from rest.

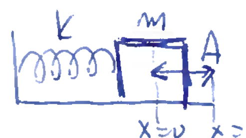
(i) What is the maximum value of the object's acceleration? (2)

(ii) What is the speed of the block when it passes through the equilibrium position? (2)

(iii) What is the speed of the block when it passes through position $x = 3$ cm relative to equilibrium? (3)

(iv) At what distance from equilibrium is the kinetic energy of the mass equal to its potential energy? (3)

(a) (i) 29.2 m/s^2	(ii) 3.50 m/s	(iii) 0.52 m/s	(iv) + 0.04 m
(b) (i) 51.0 m/s^2	(ii) 1.75 m/s	(iii) 1.52 m/s	(iv) ± 0.04 m *
(c) (i) 51.0 m/s^2	(ii) 1.75 m/s	(iii) 1.22 m/s	(iv) ± 0.02 m
(d) (i) 51.0 m/s^2	(ii) 3.50 m/s	(iii) 0.52 m/s	(iv) + 0.02 m
(e) (i) 29.2 m/s^2	(ii) 3.50 m/s	(iii) 1.52 m/s	(iv) ± 0.04 m



$k = 1.70 \times 10^3$ N/m

$m = 2$ kg $\quad A = 0.06$ m.

(i) $F_{max} = kA = 102$ N. $\quad a_{max} = F_{max}/m = 51.0$ m/s^2

(ii) $v_{max} = \omega A$, $\omega = \sqrt{k/m} = 29.15$ s^{-1} $\Rightarrow v_{max} = 1.75$ m/s.

(iii) $\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{k}{m}(A^2 - x^2) \Rightarrow v = \pm\left[\frac{k}{m}(A^2 - x^2)\right]^{1/2} \Rightarrow v = \pm\omega\sqrt{A^2 - x^2} = \pm 1.52$ m/s $(x = 0.03$ m$)$

$\Rightarrow |v| = 1.52$ m/s

(iv) $\frac{1}{2}kA^2 = 2\left(\frac{1}{2}kx^2\right) \Rightarrow x^2 = A^2/2 \Rightarrow x = \pm A/\sqrt{2} = \pm 0.04$ m

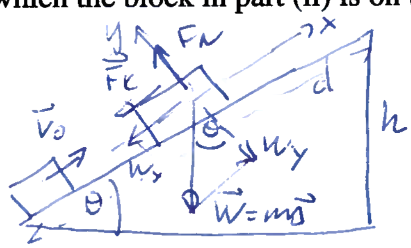
2. A 10-kg block is launched up a plane inclined at a 15° angle. The initial speed of the block is 5 m/s.

(i) How far up the inclined plane (parallel to the plane's surface) does the block slide if the coefficient of kinetic friction between the block and the surface is $\mu_k = 0.20$? (5)

(ii) Let the coefficient of static friction between the block and the surface be $\mu_s = 0.35$. In part (i), once the block stops, does it start to slide back down or does it remain at rest? Explain your reasoning. (2)

(iii) Calculate the angle of the inclined plane for which the block in part (ii) is on the verge of slipping. (3)

(a) (i) 2.82 m	(ii) Yes	(iii) 19.3°
(b) (i) 5.10m	(ii) No	(iii) 30.0°
(c) (i) 2.82 m	(ii) Yes	(iii) 19.3°
(d) (i) 1.91 m	(ii) Yes	(iii) 30.0°
(e) (i) 2.82 m	(ii) No	(iii) 19.3° *



$F_k = \mu_k F_N$

$F_N = W_y = mg\cos\theta$

$W_x = mg\sin\theta$.

$\Rightarrow F_k = \mu_k mg\cos\theta$

(i) $KE - KE_0 = W^{net} \Rightarrow KE_0 = -W^{net} = -(W_g + W_{F_N} + W_{F_k})$

$W_g = -mgh = -mgd\sin\theta \qquad W_{F_N} = 0 \qquad W_{F_k} = -F_k d = -\mu_k mgd\cos\theta$

$\Rightarrow KE_0 = -(-mgd\sin\theta - \mu_k mgd\cos\theta) \Rightarrow \frac{1}{2}mv_0^2 = mgd(\sin\theta + \mu_k\cos\theta)$

$\Rightarrow d = \dfrac{v_0^2}{2g(\sin\theta + \mu_k\cos\theta)} \Rightarrow d = 2.82$ m

(ii)

$F_s = \mu_s F_N = \mu_s mg\cos\theta$

$W_x = mg\sin\theta$

$\mu_s\cos\theta = 0.338$

$\sin\theta = 0.259$

so $F_s > W_x$

$\Rightarrow$ NO sliding

(OVER)