Drake Physics Prize Exam 2020 ... Part 2

Name: **School:** **Proctor:**

All your work must be on the problem page or on the back of that page.
Two significant figures are sufficient for the numerical answers.

1. A solid sphere of mass M with radius R and moment of inertia $I = (2/5)MR^2$ rolls down an inclined plane of height $h = 3.0\,\mathrm{m}$. If friction (except to ensure that the sphere is rolling rather than just sliding) is neglected and $h \gg R$, calculate the speed of the sphere's center of mass when reaching the bottom.

$$Mgh = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} \cdot \frac{2}{5} MR^2 \omega^2$$

$$\underset{\uparrow \atop \omega = \frac{v_{CM}}{R}}{=} \frac{1}{2} \cdot \frac{7}{5} M v_{CM}^2$$

$$\Rightarrow v_{CM} = \sqrt{\frac{10}{7} g h} \approx 6.5 \, \frac{m}{s}$$

2. Even though they attract each other, the Earth does not fall into the Sun due the conservation of angular momentum. Suppose somebody were able to stop both objects and release them from rest at an original distance of 1.5×10^{11} m. In the coordinate system where both are originally at rest, calculate the speed of the Earth when they collide.

Hints: The collision occurs when the distance between their centers is the sum of the two radii, which is very small compared to the original separation. Assume that the mass of the two objects is homogeneously distributed over their volume. Finally, note that the mass of the Earth is much smaller than that of the Sun. All these facts allow you to make approximations that will get you the result to an acceptable degree of accuracy. [Recall that two significant figures are sufficient.]

energy conservation: loss in P.E. = gain in K.E.

$$G\underbrace{\frac{M_S \cdot M_E}{(R_S + R_E)}}_{\approx R_S} - \underbrace{G\frac{M_S \cdot M_E}{1.5 \cdot 10^{11}\,m}}_{\approx 0} = \frac{1}{2} M_E v_E^2 + \underbrace{\frac{1}{2} M_S v_S^2}_{\approx 0}$$

linear momentum conservation: $M_E v_E = M_S v_S$

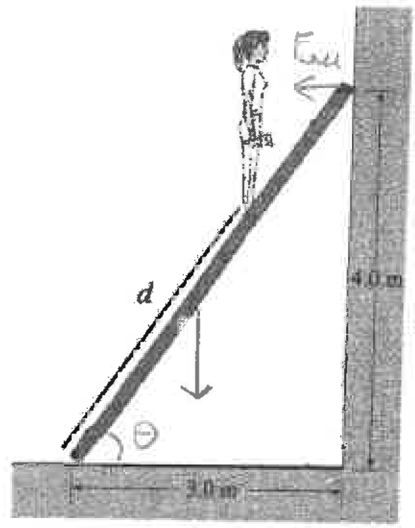
$$\Rightarrow \frac{1}{2} M_S v_S^2 = \frac{1}{2}\frac{1}{M_S}(M_S v_S)^2 = \frac{1}{2}\frac{M_E}{M_S} M_E v_E^2 << \frac{1}{2} M_E v_E^2$$

$$\Rightarrow v_E \approx \sqrt{\frac{2G \cdot M_S M_E}{M_E \cdot R_S}} = \sqrt{\frac{2GM_S}{R_S}} = 620{,}000\,\frac{m}{s}$$

$$= 620\,\frac{km}{s} = 6.2 \cdot 10^5\,\frac{m}{s}$$

This is basically the escape speed of the Earth from the surface of the Sun.

3. A person of mass $m = 60\,\text{kg}$ would like to climb up a ladder of mass $M = 20\,\text{kg}$ and length $L = 5.0\,\text{m}$. The coefficient of static friction between the bottom of the ladder and the ground is $\mu = 0.60$. Neglect the coefficient of friction between the ladder and the wall, and assume that the ladder's mass is homogeneously distributed. How far up the ladder (i.e., the distance d) can the person go before the ladder starts sliding from the position shown?



torque: $\left(Mg\frac{L}{2} + mgd\right)\cos\Theta = F_{wall}\, L \sin\Theta$

$F_W = \mu F_{normal} = \mu (M+m) g$

$\Rightarrow M\frac{L}{2} + md = \mu (M+m) L \cdot \tan\Theta$ — $\frac{4}{3}$ from the figure (note that g cancels)

$$\Rightarrow d = \frac{\mu (M+m) L \cdot \frac{4}{3} - M\frac{L}{2}}{m} = \frac{0.6 \cdot 80 \cdot 5 \cdot \frac{4}{3} - 50}{60} \text{ meters}$$

$= 4.5$ meters $\hat{=}$ not quite to the top

4. A mass $m_1 = 5.0\,\mathrm{kg}$ with a speed of $10.0\,\mathrm{m/s}$ hits a second mass $m_2 = 10.0\,\mathrm{kg}$ originally at rest. The mass m_1 is scattered (deflected) by an angle of 25° from its original direction. If the collision is purely elastic, calculate the speed of m_1 after the collision.

Note: Since $m_1 \neq m_2$, the angle between the direction after the collision is not 90°

momentum conservation: $x: \; m_1 v_1 = m_1 v_1' \cos\theta_1 + m_2 v_2' \cos\theta_2$ (1)
(always) $y: \; 0 = m_1 v_1' \sin\theta_1 - m_2 v_2' \sin\theta_2$ (2)

kinetic energy conservation: $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$ (3)
(elastic only)

Isolating the $\sin\theta_2'$ and $\cos\theta_2'$ terms in (1) and (2), then squaring and adding up allows the elimination of θ_2

$$\underbrace{(m_1 v_1 - m_1 v_1' \cos\theta_1)^2}_{m_1^2 v_1^2 - 2 m_1^2 v_1 v_1' \cos\theta_1} + m_1^2 v_1'^2 \sin^2\theta_1 = m_2^2 v_2'^2$$

$$m_1^2 v_1^2 - 2 m_1^2 v_1 v_1' \cos\theta_1 + m_1^2 v_1'^2 = m_2^2 v_2'^2$$

Now we get $m_2^2 v_2'^2$ from (3) and obtain

$$m_1^2 v_1^2 - 2 m_1^2 v_1 \cos\theta_1 \, v_1' + m_1^2 v_1'^2 = m_1 m_2 v_1^2 - m_1 m_2 v_1'^2$$

$\hat{=}$ quadratic equation in v_1'. Putting the numbers in yields

$$25 \cdot 100 - 2 \cdot 25 \cdot 10 \cdot 0.906\, v_1' + 25 \cdot v_1'^2 = 50 \cdot 100 - 50 \cdot v_1'^2$$

and collecting

$$75\, v_1'^2 - 500 \cdot 0.906\, v_1' - 2500 = 0 \quad \text{for } v_1' \text{ in } \frac{m}{s}$$

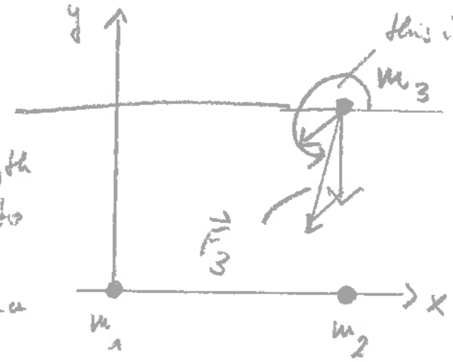
$$\Rightarrow \; v_1'^2 - 6.04\, v_1' - 33.3 = 0$$ only + makes sense

$$\Rightarrow \; v_1' = \left(3.02 \pm \sqrt{3.02^2 + 33.3}\right) \frac{m}{s} \approx 9.5 \frac{m}{s}$$

5. A mass m_1 is located at the origin of the coordinate system $(x, y) = (0, 0)$, while a second identical mass $m_2 = m_1$ is located at $(0.6, 0.0)$. Suppose that all masses interact according to Newton's Law of Gravitation, i.e., the force between two masses is attractive, proportional to the product of the masses, and inversely proportional to the distance between them. Based on this information, calculate the angle that the force due to these two masses acting on a third mass located at $(0.6, 0.6)$ makes with respect to the x-axis.

this is the angle we are looking for

F_{31} is on the diagonal, its length is $\frac{1}{2}$ of F_{32} due to the $\frac{1}{\sqrt{2}}$ in the distance



F_{32} is in $-y$ direction

on 3 due to 2

So, in terms of F_{32}, we can write the components of the force $\vec{F_3} = \vec{F}_{31} + \vec{F}_{32}$ as

$$\vec{F_3} = |\vec{F}_{32}| \cdot \begin{pmatrix} -\frac{1}{\sqrt{2}} \cdot \frac{1}{2} \\ -\frac{1}{\sqrt{2}} \cdot \frac{1}{2} - 1 \end{pmatrix} = |\vec{F}_{32}| \cdot \begin{pmatrix} -0.3536\ldots \\ -1.3536\ldots \end{pmatrix}$$

$$\Rightarrow \Theta = \arctan\left(\frac{-1.3536}{-0.3536}\right) = 255^\circ \text{ or } -105^\circ$$

watch the quadrant