Drake Physics Prize 2021

Part 1

The questions 1 – 25 are worth 1 point for the correct answer, 0 for wrong answers. You do **not** need to show how you got the solution. In some cases, you might come up with the correct answer by a good guess.

1. Which of the following is an accurate statement?

a) The magnitude of a vector can be less than the magnitude of one of its components.
b) If the magnitude of vector A is less than that of vector B, the x-component of A is smaller than the x-component of B.
c) The magnitude of a vector can be positive or negative.
d) The x-component of a vector is either zero or positive.
[✓ e)] A vector cannot have zero magnitude if one of its components is not zero.

2) A centripetal force needs to act on an object to make it travel a circular path. If the angular speed is constant, does this force do any work?

a) Yes, since a force acts and the object moves.
b) Yes, since it takes energy to move an object.
c) No, because the object has constant lateral velocity.
d) No, because the force and the displacement of the object are in opposite directions.
[✓ e)] No, because the force and the change in displacement of the object are perpendicular to each other.

3) If a constant torque is applied to an object, the object will

a) rotate with constant angular speed
[✓ b)] rotate with constant angular acceleration
c) decrease its moment of inertia
d) increase its moment of inertia
e) do none of the above

4) A car's speed decreases from 60 m/s to 30 m/s over a distance of 100 m. The average acceleration is about

a) -5.0 m/s^2
b) -9.6 m/s^2
[✓ c)] -13.5 m/s^2
d) -18.2 m/s^2
e) -23.1 m/s^2

$$v_f^2 = v_i^2 + 2ad$$

$$\Rightarrow a = \frac{3600 - 900}{200} \frac{m}{s^2} \approx -13.5 \frac{m}{s^2}$$

5) A 0.80 kg mass is attached to the end of a 1.2 m string. The system is whirled in a nearly horizontal circular path. If the maximum tension that the string can withstand is 300 N, what is the approximate maximum speed of the mass for the string not to break?

a) 5 m/s
b) 8 m/s
c) 12m/s
[✓ d)] 21 m/s
e) 26 m/s

$$F = m \frac{v^2}{r}$$

$$\Rightarrow v_{max} = \sqrt{\frac{r F_{max}}{m}} \approx 21 \frac{m}{s}$$

6) A mass hanging from a spring of negligible mass is oscillating up and down. When a second mass twice as large as the first one is added to the latter,

a) the frequency of the oscillations increases by a factor of about 3
b) the frequency of the oscillations increases by a factor of about 1.7
c) the frequency of the oscillations remains unchanged
[✓ d)] the frequency of the oscillations decreases by a factor of about 1.7
e) the frequency of the oscillations decreases by a factor of about 3

$\omega = \sqrt{\frac{k}{m}}$

$\Rightarrow m \rightarrow 3m$

$\Rightarrow \omega \rightsquigarrow \frac{\omega}{\sqrt{3}} \approx \frac{\omega}{1.7}$

7) In an introductory lab, you measure the length and width of a rectangle as 1.125 m and 0.57 m, respectively. You should report the area of that rectangle as

a) 0.6 m^2
[✓ b)] 0.64 m^2
c) 0.641 m^2
d) 0.6413 m^2
e) 0.64125 m^2

two significant figure

8) Consider two spherical planets with homogeneous mass distributions. Planet A has three times the mass and twice the radius of planet B. The ratio of gravitational forces experienced by objects on the surface of planet A to that experienced by objects on the surface of planet B is about

a) 1/4
b) 1/2
[✓ c)] 3/4
d) 1
e) 3/2

$g = \frac{GM}{r^2} \Rightarrow \frac{g_A}{g_B} = \frac{M_A}{M_B} \cdot \frac{r_B^2}{r_A^2} = 3 \cdot \frac{1}{2^2} = 3/4$

9) A wire is found to break when exposed to a minimum tension of 40 N. A similar wire with twice the diameter is expected to break at a minimum tension of

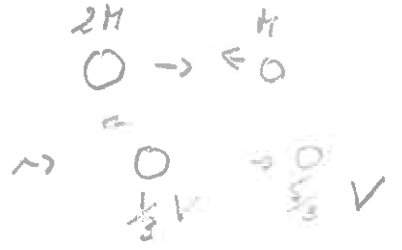
a) 40 N
b) 80 N
[✓ c)] 160 N
d) 320 N
e) 640 N

stress $\propto \frac{1}{A} \Rightarrow d \rightarrow 2d$

$\Rightarrow A \rightarrow 4A$

10) A object with mass 2M moving with velocity **V** collides head-on with an object of mass M moving with velocity –**V**. The collision is elastic, and there is no friction. What is the speed of the object with mass M after the collision?

a) zero
b) 1/2 V
c) 2/3 V
[✓ d)] 5/3 V
e) 2 V



because relative velocity has same magnitude but opposite sign

CM velocity conserved

L before: $\frac{2 \cdot V - 1V}{3} = \frac{2}{3}V$, after $\frac{-\frac{1}{3}\cdot 2V + \frac{5}{3}V}{3} = \frac{1}{3}V$ ✓

11) The quantity force times distance over time corresponds to

a) the kinetic energy of an object
b) the potential energy of an object
c) the acceleration of an object
d) the work done on an object
[✓ e) the power given to an object]

$\hat{=} \frac{\text{work}}{\text{time}} = \text{power}$

12) Which of the following statements is correct?

a) A particle's speed and velocity can be in the same direction.
b) A particle's angular momentum and velocity are in the same direction.
[✓ c) A particle's velocity and acceleration can be in opposite directions.]
d) The kinetic energy of a particle is the same as the kinetic energy of its center of mass.
e) If a force is applied to a particle, the kinetic energy will increase.

13) A canon ball is launched horizontally from a cliff with a speed of 50 m/s. Four seconds later, the ball hits the ground. Neglect air resistance. The approximate height of the cliff is

a) 30 m
b) 40 m
c) 50 m
d) 65 m
[✓ e) 80 m]

The horizontal component doesn't matter. The vertical drop is
$d = \frac{1}{2} g t^2 = (5 \cdot 4^2)\,\text{m} = 80\,\text{m}$

14) A boy and a girl, both of the same mass and general body shape, are riding a merry-go-round. The boy sits at the outer edge of the merry-go-round, whereas the girl sits half way between the center and the edge. Which of the following statements is correct?

[✓ a) Both have the same angular speed, but the boy has the larger linear speed.]
b) Both have the same angular speed, but the girl has the larger linear speed.
c) Both have the same linear speed, but the boy has the larger angular speed.
d) Both have the same linear speed, but the girl has the larger angular speed.
e) Both have the same angular and linear speed.

15) Three masses, $m_1 = 20$ kg, $m_2 = 10$ kg, and $m_3 = 30$ kg are located at the following locations on a straight line: m_1 at $= -2.0$ meter; m_2 at +3.0 meter; m_3 at + 5.0 meter. Consequently, the center of mass of this system of three masses is located at

a) 0.0 meter
b) 1.1 meter
[✓ c) 2.3 meter]
d) 4.0 meter
e) 5.2 meter

$r_{cm} = \frac{20 \cdot (-2) + 10 \cdot 3 + 30 \cdot 5}{60}\,\text{m} \approx 2.3\,\text{m}$

16) In a simple harmonic oscillator, the average kinetic energy is

[✓ a) the same as the average potential energy]
b) half the average potential energy
c) twice the average potential energy
d) the same as one third of the total energy
e) the same as three quarters of the total energy

17) Compared to yesterday, you did twice the work in two thirds of the time. To do so, your power output must have been

a) the same as yesterday
[✓ b) three times that of yesterday]
c) one third of yesterday's
d) nine times that of yesterday
e) one ninth of yesterday's

power = work/time so $\frac{\times 2}{2/3} = 3$

18) The driver of a motorboat that can move at 12 m/s in still water wishes to travel in a straight line directly across a river in which the current flows at 4.0 m/s. At what angle (relative to the straight line) should the driver head the boat?

[✓ a) 19 degrees]
b) 27 degrees
c) 45 degrees
d) 55 degrees
e) 70 degrees

$\theta = \arcsin\left(\frac{4}{12}\right) = 19°$

19) A car with mass 800 kg is going around a curve banked at 20.0 degrees. The normal force exerted by the surface on the car is

a) unknown, since neither the speed of the car nor the radius of the curve are given.
b) about 7,850 N
[✓ c) about 8,350 N]
d) about 10,600 N
e) about 12,700 N

$F_N = \frac{mg}{\cos 20°} \approx 8,350\ N > mg$

20) Two objects collide and bounce off each other. Which of the following statements is correct?

a) The kinetic energy is always conserved.
b) The kinetic energy is only conserved if the collision is head-on.
c) The "center of mass" (CM) changes from the lighter to the heavier particle.
[✓ d) The velocity of the CM before and after the collision is the same.]
e) If the collision is completely inelastic, the total linear momentum of the system is zero after the collision.

21) A bullet of mass 20 g is fired with a speed of 80 m/s into a 2.0 kg block of wood hanging on a rope of length 10 m at rest directly above the bullet. How high (approximately) will the center of mass of the block (with the bullet stuck in it) rise above the original position of the block?

a) 1.8 m
b) 2.3 m
[✓ c) 3.2 m]
d) 6.4 m
e) 8.1 m

$\frac{1}{2} m_{bullet} v_{bullet}^2 = (m_{bullet} + m_{block})\, g \cdot h$

$\Rightarrow$

$h \approx 3.2$ m

22) A centrifuge rotating at 3,000 rpm is stopped (with a constant angular deceleration) over a period of 10 seconds. About how many rotations were completed during these 10 seconds?

a) 1000
b) 600
c) 500
d) 300
e) 250

23) Alice and Bob are arguing as to whether or not it is, in principle, possible for an elevator to provide an acceleration of magnitude greater than g. They come up with the arguments listed below. Which one is correct?

a) No, because once the magnitude of the acceleration reaches g, the elevator is in free fall.
b) No, because an acceleration greater than g violates Newton's Second Law.
c) Yes, but only when the cable breaks.
d) Yes, if the motor is strong enough.
e) No, since g is the terminal acceleration on Earth.

24) A simple pendulum A swings with the twice the frequency, but half the amplitude of another simple pendulum B. Which of the following statements is correct?
a) The mass of pendulum B is twice the mass of pendulum A.
b) The mass of pendulum B is four times the mass of pendulum A.
c) The length of pendulum B is twice the length of pendulum A.
d) The length of pendulum B is four times the length of pendulum A.
e) The two pendula have the same length.

25) Which of the following is a unit for angular momentum?
a) J s
b) kg cm³/s²
c) kg cm³/s.
d) N m/s
e) kg m²/s²

$$\underset{\substack{\uparrow \\ \text{mass}}}{m} v r \mathrel{\hat{=}} kg \frac{m}{s} m$$

$$= \underbrace{\underbrace{\underbrace{kg \frac{m}{s^2}}_{N} \cdot m}_{J} \cdot s}_{Js}$$

Part 2

The questions 26 – 35 are worth 2 points for a correct answer, 0 for wrong answers. Again, you do **not** need to show in detail how you got the solution, but the number of possible answers is significantly larger than in the previous problems, so that the chances for a correct guess are reduced. Also, some of the questions are follow-ups on a previous one.

26) Consider three vectors in the (x,y,z) coordinate system. They are $\mathbf{V}_1 = (2.00, 4.00, -5.00)$, $\mathbf{V}_2 = (3.00, -5.00, 2.00)$, $\mathbf{V}_3 = (-2.00, 4.00, -2.00)$. The longest vector that you can make by adding or subtracting these three vectors in any +/– sequence has an approximate length of

a) 14.2
[✓ b) 16.1]
c) 18.1
d) 19.5
e) 21.8
f) 23.5
g) 26.8
h) 31.9
i) 34.0
j) 37.2

We need to consider the following combinations

$$+++ \;\hat{=}\; \begin{pmatrix} 3 \\ 5 \\ -3 \end{pmatrix}$$

$$++- \;\hat{=}\; \begin{pmatrix} 7 \\ -5 \\ -1 \end{pmatrix}$$

$$+-+ \;\hat{=}\; \begin{pmatrix} -3 \\ 13 \\ 9 \end{pmatrix} \rightarrow \text{This [illegible]} \;\hat{=}\; \text{length} = \sqrt{9+169+81} \approx 16.1$$

$$(+--) \;\hat{=}\; \begin{pmatrix} 3 \\ 5 \\ -5 \end{pmatrix}$$

27) The angle between the vectors $\mathbf{V}_1+\mathbf{V}_2-\mathbf{V}_3$ and $\mathbf{V}_1 - \mathbf{V}_2$ with the vectors specified in Problem 26) is about

a) 11.2 degrees
b) 19.3 degrees
c) 40.2 degrees
d) 66.0 degrees
e) 75.7 degrees
f) 96.3 degrees
g) 105 degrees
[✓ h) 117 degrees]
i) 132 degrees
j) 160 degrees

$$\Theta = \arccos\left(\frac{(V_1+V_2-V_3) \overset{\swarrow \text{scalar product}}{\cdot} (V_1-V_2)}{\underbrace{|V_1+V_2-V_3(}_{\text{magnitudes}} \underbrace{|V_1-V_2)}}\right)$$

$$= \arccos\left(\frac{-7-45+7}{\sqrt{75}\cdot\sqrt{131}}\right) \approx 117^\circ \qquad \left(\begin{array}{l}\text{Note: } \underline{\text{not}}\ 63^\circ \\ \hat{=}\ \text{watch the} \\ \text{quadrant!}\end{array}\right)$$

28) A golfer shoots the ball with an initial speed of 60.0 m/s and would like to make a "hole in one". The hole happens to be 300 m away from the position that he hits it from, and it is at the same level (i.e., neither uphill nor downhill). Neglect air resistance and other aerodynamic effects. The angle relative to the horizontal for the take-off of the ball should be about

a) 7 degrees
b) 10 degrees
c) 13 degrees
d) 18 degrees
e) 21 degrees
f) 24 degrees
[✓ g) 27 degrees]
h) 31 degrees
i) 35 degrees
j) 42 degrees

$$d = \frac{v_0^2}{g} \sin 2\Theta$$

$$\Rightarrow \Theta = \frac{1}{2} \arcsin\left(\frac{d \cdot g}{v_0^2}\right) \approx 27.4^\circ$$

29) Using the parameters and the result of Problem 28), the flight time of the ball would be about

[✓ a) 5.6 seconds]
b) 5.9 seconds
c) 6.2 seconds
d) 6.6 seconds
e) 7.3 seconds
f) 7.6 seconds
g) 8.2 seconds
h) 8.6 seconds
i) 9.3 seconds
j) 9.8 seconds

$$T = \frac{2 v_0 \sin \theta}{g} \approx 5.6 \text{ seconds}$$

30) A skier starts on top of a hill that is 100 m higher than the bottom where we measure his speed. The constant slope of the hill (relative onto the horizontal) is 18.0 degrees. The coefficient of friction between the skier and the snow on which he slides is 0.150. The speed of the skier at the bottom of the hill is approximately

a) 36 m/s
b) 38 m/s
[✓ c) 41 m/s]
d) 43 m/s
e) 46 m/s
f) 49 m/s
g) 51 m/s
h) 54 m/s
i) 56 m/s
j) 59 m/s

potential energy: mgh

loss due to friction $\mu F_N \cdot d = \mu m g \cdot \frac{100\,m}{\cos 18^\circ}$

=> available for kinetic energy at the bottom

$$mgh - mgh\mu \cdot \frac{1}{\cos 18^\circ} \Rightarrow \frac{1}{2} m v_{bottom}^2 \Rightarrow v_{bottom} = \sqrt{2gh\left[1 - \frac{0.15}{\cos 18^\circ}\right]} \approx 41 \frac{m}{s}$$

$\uparrow$ m drops out

31) Using the result and the parameters of Problem 30, the skier can slide further until friction will stop him. The extra horizontal distance that the skier can travel starting at the bottom of the hill is about

a) 110 m
b) 140 m
c) 170 m
d) 190 m
e) 240 m
f) 280 m
g) 310 m
h) 430 m
[✓ i) 560 m]
j) 700 m

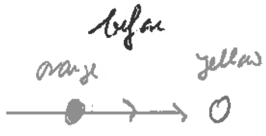
$$\frac{1}{2} m v_{bottom}^2 = \underset{\overset{\uparrow}{mg}}{F_N} \cdot d_{extra}\, \mu$$

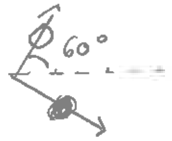
=> again the mass drops out

$$\Rightarrow d_{extra} \approx 560\,m$$ (with more accurate result from 30,
≈ 570 with $41 \frac{m}{s}$)

32) Two shuffleboard disks of equal mass, one orange and the other yellow, are involved in an elastic glancing collision. The yellow disk is initially at rest and the orange disk is moving with a speed of 5.00 m/s. After the collision, the orange disk makes an angle of 30.0 degrees relative to its initial direction, while the velocity of the yellow disk is perpendicular to that of the orange disk. The speed of the yellow disk after the collision is about

a) 3.4 m/s
b) 3.3 m/s
c) 3.2 m/s
d) 3.1 m/s
e) 3.0 m/s
f) 2.9 m/s
g) 2.8 m/s
h) 2.7 m/s
i) 2.6 m/s
[✓ j) 2.5 m/s]





momentum conservation

$v_{yellow}' \sin 60° = v_{orange} \sin 30°$

$\Rightarrow v_{yellow} = \frac{v_{orange}}{\sqrt{3}}$

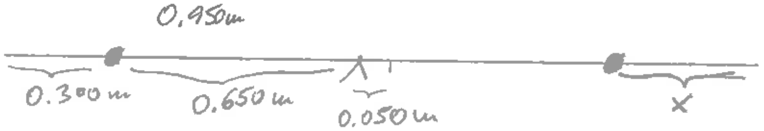
kinetic energy conservation ; $25 \frac{m^2}{s^2} = \left(\frac{1}{3} + 1\right) v_{orange} \Rightarrow v_{orange} = \sqrt{\frac{3}{4}} \cdot 5 \frac{m}{s}$
($\frac{1}{2}m$ drops out)

$\Rightarrow v_{yellow} = \frac{1}{2} 5 \frac{m}{s} = 2.5 \frac{m}{s}$

33) A seesaw consisting of a board of length 2.00 m and mass 20.0 kg (with homogeneous mass distribution) is slightly misaligned in that the pivot point is at 0.950 m to one side and 1.05 m to the other. A boy of mass 30.0 kg sits on the 0.950 m side, so that his center-of mass is located 0.300 m from the end of the board. Where should the center of mass of a 25.0 kg girl relative to the end of the 1.05 m side be placed so that the seesaw is in equilibrium? The result is closest to which of the following possibilities?

0

a) 0.21 m
b) 0.23 m
c) 0.25 m
d) 0.27 m
e) 0.29 m
[✓ f) 0.31 m]
g) 0.33 m
h) 0.35 m
i) 0.37 m
j) 0.39 m



replace the board by its CM, which 0.050m from the pivot and helps the girl

$\Rightarrow 30.0\,kg \cdot 0.650\,m = 20.0\,kg \cdot 0.050\,m + 25.0\,kg \cdot (1.050\,m - x)$

$\Rightarrow x = 0.31\,m$

34) Consider a motorcycle of mass 150 kg, one wheel of which has a mass M = 10.0 kg and a radius R = 0.300 m. Each wheel has a moment of inertia of 0.600 M R². The ratio of the total rotational energy of the motorcycle to its total kinetic energy is about

a) 0.22
b) 0.24
c) 0.26
d) 0.28
e) 0.35
f) 0.42
g) 0.48
[✓ h) 0.55]
i) 0.59
j) 0.65

rotational K.E. for each wheel is $\frac{1}{2} I \omega^2 = 0.3 M R^2 \frac{v^2}{R^2}$

$\Rightarrow$ total rotational K.E. is $2 \cdot 0.3 \cdot M v^2 = 0.6 M v^2$

$\Rightarrow$ total kinetic energy $(0.6 + \underset{\substack{\uparrow \\ \text{translational}}}{0.5}) M v^2$

$\Rightarrow$ ratio is $\frac{0.6}{1.1} \approx 0.55$

35) In the following problem, assume that the Earth is a homogeneous sphere of radius 6.370 x 10^6 m corresponding to sea level. The difference in weight of a person with a mass of 80.0 kg, when measured at sea level (assume $g = 9.81 \text{ m/s}^2$ there) and at an altitude of 5,000 m above sea level is about

a) 0.45 N
b) 0.63 N
c) 0.79 N
d) 0.92 N
[✓ e) 1.2 N]
f) 1.5 N
g) 1.8 N
h) 2.1 N
i) 2.6 N
j) 3.0 N
j) 3.4 N

$$g_h = \frac{GM}{(r_E + h)^2}$$

$$\Rightarrow \frac{g_{5000}}{g_0} = \left(\frac{6.370}{6.375}\right)^2 \approx 0.998432$$

$$\Rightarrow \left| g_{5000} - g_0 \right| \cdot m \cdot g \approx 1.23 N \approx 1.2 N$$