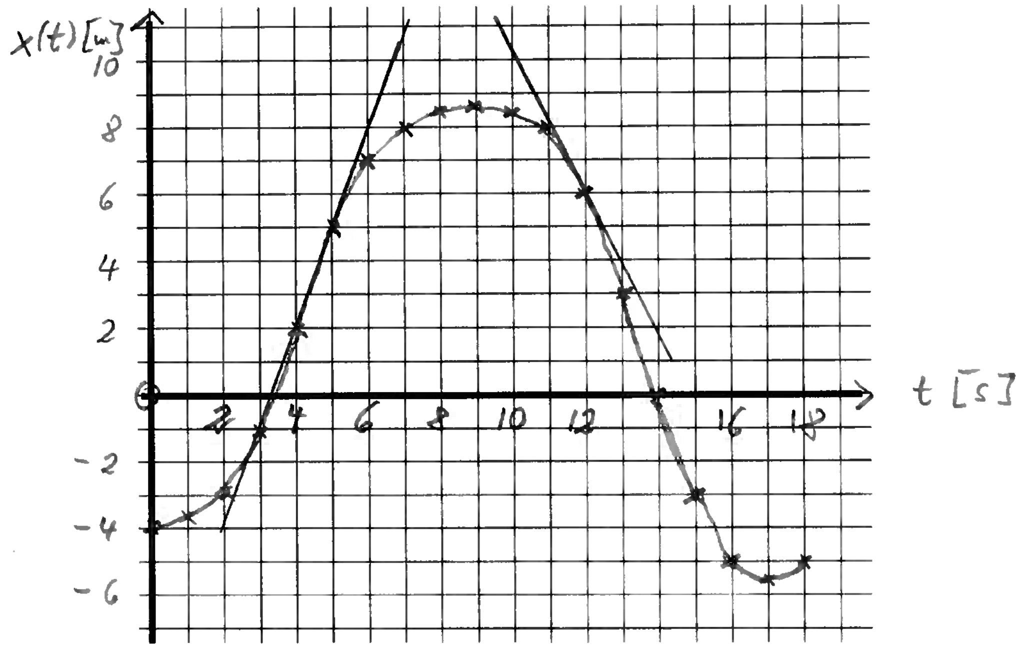
Drake Physics Prize Exam 2012 ... Part 2

1. The table below gives the position of a body as a function of time.

t[s]	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
x[m]	-4.0	-3.8	-3.0	-1.0	2.0	5.0	7.0	8.0	8.5	8.7	8.5	8.0	6.0	3.0	0.0	-3.0	-5.0	-5.5	-5.0

a) Use the graph paper below to sketch the function $x(t)$. (3 pts)



b) What are the average velocity and the average speed between $t_1 = 5\,\mathrm{s}$ and $t_2 = 15\,\mathrm{s}$? (3 pts)

$$\vec{v}_{ave} = \frac{x(15) - x(5)}{10\,s} = \frac{-3.0\,m - 5.0\,m}{10\,s} = -0.80\,\frac{m}{s}$$

$$|\vec{v}_{av}| = \frac{\text{total distance}}{10\,s} = \frac{3.7\,m + 11.7\,m}{10\,s} \approx 1.5\,\frac{m}{s}$$

c) What are the instantaneous velocities at $t_1 = 6\,\mathrm{s}$ and $t_2 = 12\,\mathrm{s}$? (3 pts)

We need the slopes.

At 5s : $\vec{v}_1 \approx \frac{5.6\,m}{2.0\,s}\,\hat{x} \approx 2.8\,\frac{m}{s}\,\hat{x}$

At 12 s : $\vec{v}_2 \approx -\frac{4.0\,m}{2.0\,s}\,\hat{x} \approx -2.0\,\frac{m}{s}\,\hat{x}$

d) What is the average acceleration between $t_1 = 6\,\mathrm{s}$ and $t_2 = 12\,\mathrm{s}$? (1 pt)

$$\vec{a}_{av} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1} \approx \frac{-4.8\,\frac{m}{s}}{6\,s}\,\hat{x} \approx -0.8\,\frac{m}{s^2}\,\hat{x}$$

2. The Earth goes around the Sun once a year, following an approximately circular path of radius 1.5×10^{11} m. In addition, the Earth rotates around its own center axis once per day. The moment of inertia for a homogeneous sphere is $\frac{2}{5} MR^2$ (approximate the Earth as such), the mass of the Earth is 6.0×10^{24} kg, and its radius is 6.4×10^6 m.

a) What is the angular speed (in rad/s) of the Earth's rotation around its axis? (2 pts)

$$\frac{2\pi\ [\text{rad}]}{24\ \text{hrs}} \approx 7.3 \times 10^{-5}\ \text{rad}/\text{s}$$

b) What is the tangential speed (in m/s) of the Earth's motion around the Sun? (2 pts)

$$\frac{2\pi \cdot 1.5 \times 10^{11}\ \text{m}}{365\ \text{days}} \approx 30{,}000\ \text{m}/\text{s}$$

c) What are the kinetic energies associated with the Earth's rotation (2 pts) and translation (1 pt)?

rotation: $\frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{2}{5} \cdot 6.0 \times 10^{24}\ \text{kg} \times (6.4 \times 10^6\ \text{m})^2 \times (7.3 \times 10^{-5}\ \frac{\text{rad}}{\text{s}})^2$

$$\approx 2.6 \times 10^{29}\ \text{J}$$

translation: $\frac{1}{2} m v^2 = \frac{1}{2} \times 6.0 \times 10^{24}\ \text{kg} \cdot (30{,}000\ \frac{\text{m}}{\text{s}})^2 \approx 2.9 \times 10^{31}\ \text{J}$

d) Calculate the mass of the Sun from the above information. (3 pts)

$$\frac{m v^2}{r} = G \cdot \frac{m M}{r^2} \Rightarrow M = \frac{v^2 r}{G} \approx 2.0 \times 10^{30}\ \text{kg}$$

Earth

3. A red ball of mass $m_1 = 1.0\,\mathrm{kg}$, moving with a speed of $5.0\,\mathrm{m/s}$ along the x-direction, hits a blue ball of mass $m_2 = 2.0\,\mathrm{kg}$, initially at rest. Students performing a laboratory experiment report the following:

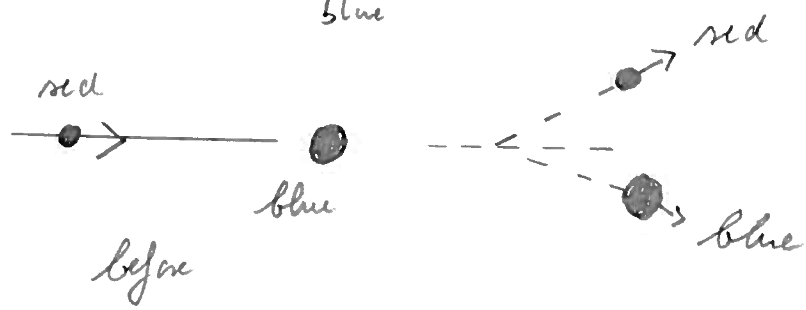
   "After the collision, the red ball moved with a speed of $3.0\,\mathrm{m/s}$ at an angle of $\theta_1 = 30°$ with respect to the x-direction."

   a) Calculate the momentum vector and the speed of the blue ball after the collision. (5 pts)

$$p_x^{blue} = m_1 v_1 - m_1 v_1' \cos 30° \approx 2.4\ \frac{kg \cdot m}{s}$$

$$p_y^{blue} = -m_1 v_1' \sin 30° \approx -1.5\ kg \frac{m}{s}$$

$$\Rightarrow v_2' = \sqrt{p_x^2 + p_y^2} \,/\, m_{blue} \approx 1.5\ m/s$$



   b) Is this an elastic or an inelastic collision? Justify your answer quantitatively. (3 pts)

before: $E_{total} = \frac{1}{2} m_1 v_1^2 \approx 12\ J$

after: $E_{total} = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \approx 6.6\ J < 12\ J$

$\Rightarrow$ clearly inelastic

4. Suppose a tuning fork vibrating at 512 Hz in air (speed of sound 340 m/s) falls from rest and accelerates down with 9.81 m/s^2.

a) Derive a general formula for the frequency heard at the release point as a function of time after the moment of release. (3 pts)

This is the case of a moving source => λ appears to be shorter when the source is approaching.

$$v = \lambda \cdot f = \lambda' f' \Rightarrow f' = f \cdot \frac{\lambda}{\lambda'} = f \cdot \frac{\lambda}{\lambda - v_s T}$$

$$= f \cdot \frac{\lambda / T}{\frac{\lambda}{T} - v_s} = f \cdot \frac{v}{v - v_s}$$

Here $v \mathrel{\hat{=}}$ speed of sound, $v_s \mathrel{\hat{=}}$ speed of the (approaching) source, T = period.

In this particular case; $v_s = \overset{\downarrow}{-} g t \Rightarrow f' = f \cdot \frac{v}{v + gt}$

b) How far below the point of release is the tuning fork, when the frequency registered at the release point is 485 Hz? (5 pts)

$$485 \text{ Hz} = 512 \text{ Hz} \cdot \frac{340}{340 + 9.81 \cdot t_{end} [s]}$$

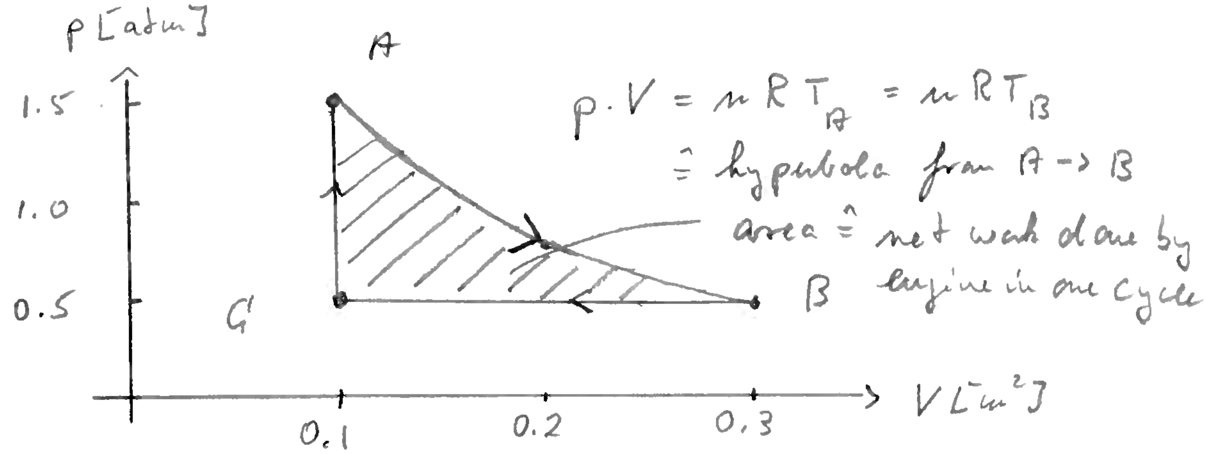
$$\Rightarrow 340 + 9.81\, t_{end} = \frac{512}{485} \cdot 340 = 359$$

$$\Rightarrow t_{end} [s] = \frac{359 - 340}{9.81} \approx 1.93 \text{ s}$$

$$\Rightarrow d = \frac{1}{2} g\, t_{end}^2 \approx 18.3 \text{ m}$$

5. Three moles of a monatomic ideal run through a reversible cycle. At the starting point (A), the pressure is 1.50 atm (150 kPa) and the volume is $0.100\,\text{m}^3$. In the first step, the gas expands isothermally to point B, with $V_B = 3V_A$. Next, it undergoes an isobaric compression back to its original volume at point C, and finally an isochoric process back to the starting point A.

   a) Sketch the cycle, including all directions, in a PV-diagram. Make sure to label the axes and add scales. Also, mark the net work done by the gas. (3 pts)



   b) Calculate the temperature at points A and C, and the pressure at point B. (3 pts)

$$T_A = \frac{p_A \cdot V_A}{3\,\text{mol} \times R} \approx 600\,K$$

Since $p \cdot V = nRT$ and $U = \frac{3}{2} nRT = \frac{3}{2} pV$, we see

$$T_C = \frac{1}{3} T_A \approx 200\,K; \quad P_B = \frac{1}{3} P_A \approx 0.5\,\text{atm}$$

   c) Calculate the heat that is added to the gas in the step C → A. (2 pts)

$$\Delta U = \frac{3}{2} nR(\Delta T) = \frac{3}{2} (\Delta p) \cdot V \approx 15{,}000\,J$$

   d) According to calculus, the work done by the gas in step A → B alone is about 16,500 J. Calculate the efficiency of the engine. (2 pts)

$$\eta = \frac{16{,}500\,J - 0.5\,\text{atm} \cdot 0.2\,m^3}{15{,}000\,J + 16{,}500\,J} \approx \frac{6{,}500}{31{,}500} \approx 21\,\%$$